Rare events in turbulence might not be so rare after all

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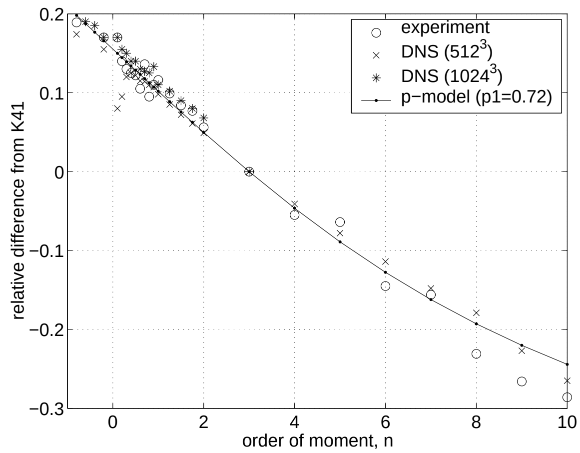
In 1941, the Russian mathematician Andrei N. Kolmogorov deduced scaling laws for the velocity statistics of turbulent flows. At the core of the Kolmogorov theory lies the hypothesis that the scales in fully developed turbulence are statistically *self-similar*. The theory predicts that the n-th order moment of the velocity difference across scales of size r should scale as $r^{n/3}$. Experimental data has shown that for $n > 3$ the scaling exponents are $\zeta_n < n/3$. It is now believed that the turbulent scales are not self-similar but *intermittent* and the scaling exponents of moments of velocity increments are *anomalous*, that is, the departures from Kolmogorov's self-similar scaling increase nonlinearly with the increasing order of the moment. Since high-order moments sample the tails of a probability distribution function, it is also believed that the intermittency in turbulence is associated with rare events. We have shown, using a compilation of data from experiments and direct numerical simulations, that intermittency is not merely associated with the rare events in the flow but is in fact present in the high frequency events. Thus our results motivate the theoretical study of anomalous scaling in the limit of zeroth order moments, unbiased by so-called rare events.

The moments of the velocity difference across spatial scales are known as structure functions and are defined by

$$\begin{aligned} S_n(r) &= \langle|\delta u(r)|^n\rangle \\ &= \int_0^\infty |\delta u(r)|^n P(\delta u(r)) d(\delta u(r)) \quad (1) \end{aligned}$$

where $\delta u(r) = (\boldsymbol{u}(\boldsymbol{x}+\boldsymbol{r}) - \boldsymbol{u}(\boldsymbol{x}))\cdot\hat{\boldsymbol{r}}$ is the longitudinal velocity increment across scales of size r. The $\langle\cdot\rangle$ denotes an ensemble average which in practice is implemented as an average over the space-time domain. $P(\delta u(r))$ is the probability density distribution. In the probablistic sense, it is clear from the above equation that the large-n moments are associated with tail of P, or the low probability events. Structure functions are useful measures of the statistical properties of turbulent flows as a function of scale. In particular, the second-order moment of velocity difference across scales of size r is a measurement of the energy contained in those scales.



The relative departure from the Kolmogorov 1941 self-similarity theory for moments ranging from $-0.8 \le n \le 10$ as computed from various sources of data. The solid line is the comparison to the so-called p-model [1] derived by assuming multifractality of the scales. The important feature to note is that for $n < 2$ the absolute relative departure from Kolmogorov scaling increases at the same rate as for $n > 2$. This means that intermittency, the departure from the Kolmogorov self-similarity scaling is not confined to just a higher-order moments, or to the rare events

Kolmogorov derived an exact law from the energy balance equation governing the third-order structure function, yielding $S_3 \sim r$ for homogeneous and isotropic turbulence. The self-similarity assumption means, for example, that the energy in scales of size λr would be simply be the energy in scales of size r scaled simply by a fixed power of λ. Equivalent rules govern moments of arbitrary order. Together these results yield for statistically isotropic turbulence:

$$S_n(r) \sim r^{n/3} \quad (2)$$

Empirical measurement of self-similar scaling for $n = 2$ and 3 have been consistent with the Kolmogorov theory. However, as higher-order moments began to be reliably measured, it became clear that the there is a pronounced departure from the self-similar scaling laws.

We analyzed three sets of data in this work. Experimental measurements of velocity fluctuations in the atmospheric boundary layer, resolved direct numerical simulations (DNS) of the forced Navier-Stokes equation in a periodic cube of 512 grid-point to a side, and another computed in a cube of 1024 grid points to a side. The small-scale Reynolds number ranged 200 (smaller DNS) to 10,000 (experiments) over the datasets. For the larger DNS we removed an element of uncertainty by spherically averaging the structure functions at each order over the sphere so that anisotropic effects, if any should be present, were projected out. The scheme used in the spherical averaging procedure is described in [2].

A rigorous theory to explain anomalous scaling and intermittency in turbulence does not yet exist. Most current efforts are focussed on the intermittency for large n. The empirical results presented here provide motivation to seek a theoretical explanation for intermittency in the high frequency events, and correspondingly for anomalous scaling in the low-order moments.

This report is a summary of work published in [3].

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